Module 10: Time Independent Perturbation Theory

10.1 Consider a 4-fold degenerate state with orthonormal eigenfunctions \( u_1, u_2, u_3 \) and \( u_4 \). There is a perturbation \( H' \). It is given that \( H'_{12} = H'_{21} = -g \); \( g > 0 \) and all the other matrix elements are zero. Find the splitting and corresponding wavefunctions.

(a) \( g, -g, 0, 0 \)
(b) \( 2g, g, 0, 0 \)
(c) \( g, g, 0, 0 \)
(d) \( g, 0, 0, 0 \)

[Answer (a)]

10.2 Consider a 4-fold degenerate state with orthonormal eigenfunctions \( u_1, u_2, u_3 \) and \( u_4 \). There is a perturbation \( H' \). It is given that \( H'_{12} = H'_{21} = -g \); \( g > 0 \), and all the other matrix elements are zero. Find the wave functions of the split levels.

(a) \( u_1, u_2, u_3 \) and \( u_4 \)
(b) \( \frac{u_1 - u_2}{\sqrt{2}}, \frac{u_1 + u_2}{\sqrt{2}}, u_3 \) and \( u_4 \)
(c) \( \frac{u_1 - 2u_2}{\sqrt{2}}, \frac{u_1 + 2u_2}{\sqrt{2}}, u_3 \) and \( u_4 \)
(d) \( \frac{u_1 - 3u_2}{\sqrt{2}}, \frac{u_1 + 3u_2}{\sqrt{2}}, u_3 \) and \( u_4 \)

[Answer (b)]

10.3 Consider a 4-fold degenerate state with orthonormal eigenfunctions \( u_1, u_2, u_3 \) and \( u_4 \). There is a perturbation \( H' \). It is given that \( H'_{11} = H'_{22} = 2g \), \( H'_{12} = H'_{21} = g \) and all the other matrix elements are zero. Find the splitting and corresponding wavefunctions.

(a) \( g, g, 0, 0 \)
(b) \( 2g, g, 0, 0 \)
(c) \( 3g, g, 0, 0 \)
(d) \( 4g, g, 0, 0 \)

[Answer (c)]
10.4 Consider a 4-fold degenerate state with orthonormal eigenfunctions $u_1, u_2, u_3$ and $u_4$. There is a perturbation $H'$. It is given that $H'_{11} = H'_{22} = 2g$, $H'_{12} = H'_{21} = g$ and all the other matrix elements are zero. Find the wave functions of the split levels.

(a) $u_1, u_2, u_3$ and $u_4$
(b) $\frac{u_1 - u_2}{\sqrt{2}}, \frac{u_1 + u_2}{\sqrt{2}}, u_3$ and $u_4$
(c) $\frac{u_1 - 2u_2}{\sqrt{2}}, \frac{u_1 + 2u_2}{\sqrt{2}}, u_3$ and $u_4$
(d) $\frac{u_1 - 3u_2}{\sqrt{2}}, \frac{u_1 + 3u_2}{\sqrt{2}}, u_3$ and $u_4$

[Answer (b)]