Module 6: Hydrogen Atom and Other Two Body Problem

6.1 For a spherically symmetric potential, the radial part of the Schrodinger equation is given by:

\[ \frac{d^2R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + \frac{2\mu}{\hbar^2} \left[ E - V - l(l+1)\frac{1}{2\mu r^2} \right] R = 0. \]

The function \( F_r \) is given by

(a) \( - \frac{l + l+1}{2\mu r^2} \)
(b) \( \frac{l + l+1}{2\mu r^2} \)
(c) \( -l + l+1 \)
(d) \( + l + l+1 \)

[Answer (a)]

6.2 In the hydrogen atom problem the radial part of the Schrodinger equation can be written in the form

\[ \frac{1}{\rho^2} \frac{d}{d\rho} \left( \rho^2 \frac{dR}{d\rho} \right) + \left( \frac{\lambda}{\rho} - \frac{1}{4} - \frac{l + l+1}{\rho^2} \right) R = 0 \]

where \( \rho = \gamma r \). The quantity \( \gamma \) is given by

(a) \( \left( \frac{2\mu E}{\hbar^2} \right)^{1/2} \)
(b) \( \left( \frac{2\mu E}{\hbar^2} \right)^{-1/2} \)
(c) \( \left( \frac{8\mu E}{\hbar^2} \right)^{1/2} \)
(d) \( \left( \frac{8\mu E}{\hbar^2} \right)^{-1/2} \)

[Answer (d)]

6.3 In the hydrogen atom problem the radial part of the Schrodinger equation can be written in the form
\[
\frac{1}{\rho^2} \frac{d}{d\rho} \left( \rho^2 \frac{dR}{d\rho} \right) + \left( \frac{\lambda}{\rho} - \frac{1}{4} - \frac{l(l+1)}{\rho^2} \right) R = 0
\]

where \( \rho = \gamma r \). The quantity \( \lambda \) is given by

(a) \( Z\alpha \left( -\frac{\mu c^2}{8E} \right)^{1/2} \)

(b) \( Z\alpha \left( \frac{\mu c^2}{8E} \right)^{1/2} \)

(c) \( Z\alpha \left( -\frac{\mu c^2}{2E} \right)^{1/2} \)

(d) \( Z\alpha \left( +\frac{\mu c^2}{2E} \right)^{1/2} \)

where \( \alpha \) is the fine structure constant.

[Answer (c)]

6.4 In the hydrogen atom problem, the radial part of the Schrodinger equation can be written in the from

\[
R_{nl} \rho = N\rho^l e^{-\rho/2} \, \, _1F_1 \, a, c, \rho
\]

where \( _1F_1 \, a, c, \rho \) is the confluent hypergeometric function. The infinite series \( _1F_1 \, a, c, \rho \) must be made into a polynomial because \( _1F_1 \, a, c, \rho \)

(a) is a divergent series for all values of \( \rho \).
(b) is a divergent series only for \( \rho > 1 \)
(c) is a convergent series but behaves \( \rho^{a-c} e^{+p/3} \) as \( \rho \to \infty \).
(d) is a convergent series but behaves \( \rho^{3a-c} e^{+a} \) as \( \rho \to \infty \).

[Answer (d)]

6.5 In the hydrogen atom problem, the radial part of the Schrodinger equation can be written in the from

\[
R_{nl} \rho = N\rho^l e^{-\rho/2} \, \, _1F_1 \, a, c, \rho
\]
where \( \mathbf{F}_1 a,c,\rho \) is the confluent hypergeometric function. The infinite series \( \mathbf{F}_1 a,c,\rho \) becomes a polynomial

(a) when \( a \) becomes a positive integer
(b) when \( a \) becomes a negative integer
(c) when \( c \) becomes a positive integer
(d) when \( c \) becomes a negative integer

[Answer (b)]

6.6 In the hydrogen atom problem, the radial part of the Schrodinger equation can be written in the form

\[ R_n^l \rho = N\rho^l e^{-\rho^2/2} \mathbf{F}_1 a,c,\rho \]

where \( \mathbf{F}_1 a,c,\rho \) is the confluent hypergeometric function. The quantity \( a \) is given by

(a) \( a = l+1-n \)
(b) \( a = n-l+1 \)
(c) \( a = 2l+1 \)
(d) \( a = 2l+2 \)

where \( n \) is the total quantum number.

[Answer (a)]

6.7 In the hydrogen atom problem, the radial part of the Schrodinger equation can be written in the form

\[ R_n^l \rho = N\rho^l e^{-\rho^2/2} \mathbf{F}_1 a,c,\rho \]

where \( \mathbf{F}_1 a,c,\rho \) is the confluent hypergeometric function. The quantity \( c \) is given by

(a) \( c = l+1-n \)
(b) \( c = n-l+1 \)
(c) \( c = 2l+1 \)
(d) \( c = 2l+2 \)

where \( n \) is the total quantum number.

[Answer (d)]